Diploma Programme
Programme du diplôme
Programa del Diploma

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Mathematics: applications and interpretation
Standard level
Paper 1

Monday 31 October 2022 (afternoon)

Candidate session number

1 hour 30 minutes

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Answer all questions.
- Answers must be written within the answer boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: applications and interpretation formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[80 marks]**.

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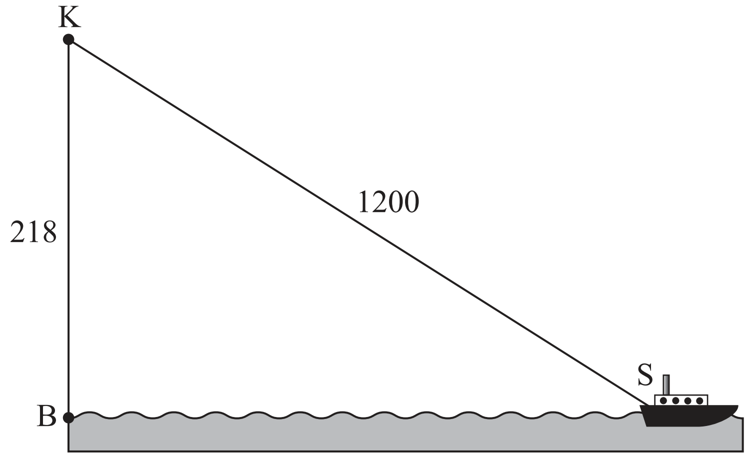
Answers must be written within the answer boxes provided. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. [Maximum mark: 6]

Kacheena stands at point K, the top of a 218 m vertical cliff. The base of the cliff is located at point B. A ship is located at point S, 1200 m from Kacheena.

This information is shown in the following diagram.

diagram not to scale



(a) Find the angle of elevation from the ship to Kacheena. [2]

(b) Find the horizontal distance from the base of the cliff to the ship. [2]

(c) Write down your answer to part (b) in the form $a \times 10^k$ where $1 \le a < 10$ and $k \in \mathbb{Z}$. [2]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 7]

In the first month of a reforestation program, the town of Neerim plants 85 trees. Each subsequent month the number of trees planted will increase by an additional 30 trees.

The number of trees to be planted in each of the first three months are shown in the following table.

Month	**Trees planted**
1	85
2	115
3	145

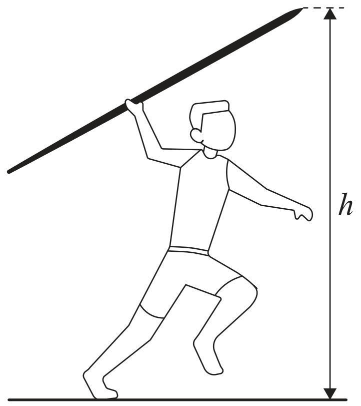
(a) Find the number of trees to be planted in the 15th month. [3]

(b) Find the total number of trees to be planted in the first 15 months. [2]

(c) Find the mean number of trees planted per month during the first 15 months. [2]

3. [Maximum mark: 5]

DeVaughn throws a javelin in a school track and field competition.



The height, h, of the front tip of the javelin above the ground, in metres, is modelled by the following quadratic function,

$$h(t) = -3.6t^2 + 10.8t + 1.8,\ t \geq 0$$

where t is the time in seconds after the javelin is thrown.

(a) Write down the height of the front tip of the javelin at the time it is thrown. [1]

(b) Find the value of t when the front tip of the javelin reaches its maximum height. [2]

(c) Find the value of t when the front tip of the javelin strikes the ground. [2]

4. [Maximum mark: 5]

Sergio is interested in whether an adult's favourite breakfast berry depends on their income level. He obtains the following data for 341 adults and decides to carry out a χ^2 test for independence, at the 10% significance level.

		Income level		
		Low	**Medium**	**High**
Favourite berry	**Strawberry**	21	39	30
	Blueberry	39	67	42
	Other berry	32	45	26

(a) Write down the null hypothesis. [1]

(b) Find the value of the χ^2 statistic. [2]

The critical value of this χ^2 test is 7.78.

(c) Write down Sergio's conclusion to the test in context. Justify your answer. [2]

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5. [Maximum mark: 6]

Celeste heated a cup of coffee and then let it cool to room temperature. Celeste found the coffee's temperature, T, measured in $^{\circ}\mathrm{C}$, could be modelled by the following function,

$$T(t) = 71\,\mathrm{e}^{-0.0514t} + 23\,,\; t \geq 0\,,$$

where t is the time, in minutes, after the coffee started to cool.

(a) Find the coffee's temperature 16 minutes after it started to cool. [2]

The graph of T has a horizontal asymptote.

(b) Write down the equation of the horizontal asymptote. [1]

(c) Write down the room temperature. [1]

(d) Given that $T^{-1}(50) = k$, find the value of k. [2]

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6. [Maximum mark: 5]

Manny and Annabelle, mathematics teachers at Burnham High School, give their students the same examination. A random sample of the examination scores were collected from each of their classes.

Examination scores from Manny's class	76	77	82	84	88	90	91	98
Examination scores from Annabelle's class	68	79	81	89	91	92	92	95

Annabelle uses these scores to conduct a two-tailed t-test to compare the means of the two classes, at the 5% level of significance. It is assumed the examination scores for both classes have the same variance and are normally distributed.

The null hypothesis is $\mu_1 = \mu_2$, where μ_1 is the mean examination score from Manny's class and μ_2 is the mean examination score from Annabelle's class.

(a) Write down the alternative hypothesis. [1]

(b) Find the p-value for this test. Give your answer correct to five decimal places. [2]

Annabelle concludes there is insufficient evidence to reject the null hypothesis.

(c) State whether Annabelle's conclusion is correct. Give a reason for your answer. [2]

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7. [Maximum mark: 7]

On 1 December 2022, Laviola invests 800 euros (EUR) into a savings account which pays a nominal annual interest rate of 7.5% compounded monthly. At the end of each month, Laviola deposits an additional $\text{EUR } 500$ into the savings account.

At the end of k months, Laviola will have saved enough money to withdraw $\text{EUR } 10\,000$.

(a) Find the smallest possible value of k, for $k \in \mathbb{Z}^+$. [4]

(b) For this value of k, find the interest earned in the savings account. Express your answer correct to the nearest EUR. [3]

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8. [Maximum mark: 5]

Roy is a member of a motorsport club and regularly drives around the Port Campbell racetrack.

The times he takes to complete a lap are normally distributed with mean 59 seconds and standard deviation 3 seconds.

(a) Find the probability that Roy completes a lap in less than 55 seconds. [2]

Roy will complete a 20 lap race. It is expected that 8.6 of the laps will take more than t seconds.

(b) Find the value of t. [3]

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9. [Maximum mark: 7]

Taizo plays a game where he throws one ball at two bottles that are sitting on a table. The probability of knocking over bottles, in any given game, is shown in the following table.

Number of bottles knocked over	0	1	2
Probability	0.5	0.4	0.1

(a) Taizo plays two games that are independent of each other. Find the probability that Taizo knocks over a **total** of two bottles. [4]

In any given game, Taizo will win k points if he knocks over two bottles, win 4 points if he knocks over one bottle and lose 8 points if no bottles are knocked over.

(b) Find the value of k such that the game is fair. [3]

10. [Maximum mark: 6]

Stars are classified by their brightness. The brightest stars in the sky have a magnitude of 1. The magnitude, m, of another star can be modelled as a function of its brightness, b, relative to a star of magnitude 1, as shown by the following equation.

$$m = 1 - 2.5\log_{10}(b)$$

The star called Acubens has a brightness of 0.0525.

(a) Find the magnitude of Acubens. [2]

Ceres has a magnitude of 7 and is the least bright star visible without magnification.

(b) Find the brightness of Ceres. [2]

(c) Find how many times brighter Acubens is compared to Ceres. [2]

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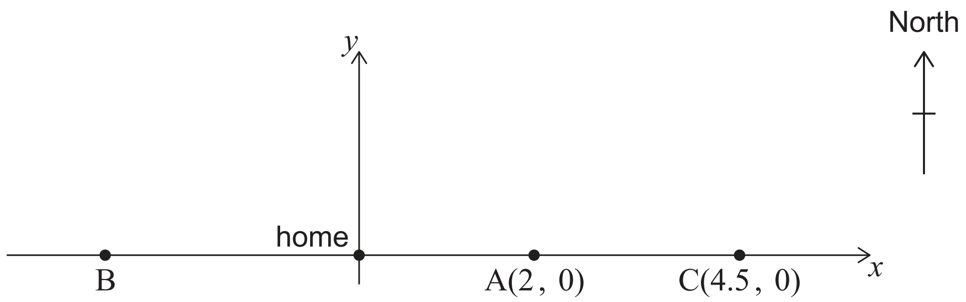
11. [Maximum mark: 7]

Kristi's house is located on a long straight road which traverses east–west. The road can be modelled by the equation $y = 0$, and her home is located at the origin $(0, 0)$.

She is training for a marathon by running from her home to a point on the road and then returning to her home by bus.

- The first day Kristi runs 2 kilometres east to point A$(2, 0)$.
- The second day Kristi runs west to point B.
- The third day Kristi runs 4.5 kilometres east to point C$(4.5, 0)$.

This information is represented in the following diagram.



Each day Kristi increases the distance she runs. The point she reaches each day can be represented by an x-coordinate. These x-coordinates form a geometric sequence.

(a) Show that the common ratio, r, is -1.5. [2]

On the 6th day, Kristi runs to point F.

(b) Find the location of point F. [2]

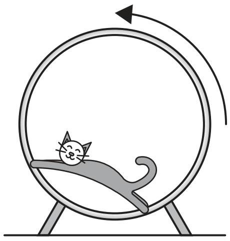
(c) Find the total distance Kristi runs during the first 7 days of training. [3]

(This question continues on the following page)

(Question 11 continued)

12. [Maximum mark: 6]

A cat runs inside a circular exercise wheel, making the wheel spin at a constant rate in an anticlockwise direction. The height, h cm, of a fixed point, P, on the wheel can be modelled by $h(t) = a\sin(bt) + c$ where t is the time in seconds and a, b, $c \in \mathbb{R}^{+}$.



When $t = 0$, point P is at a height of 78 cm.

(a) Write down the value of c. [1]

When $t = 4$, point P first reaches its maximum height of 143 cm.

(b) Find the value of

(i) a.

(ii) b. [3]

(c) Write down the minimum height of point P. [1]

Later, the cat is tired, and it takes twice as long for point P to complete one revolution at a new constant rate.

(d) Write down the new value of b. [1]

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13. [Maximum mark: 8]

Giles charges a customer per hour to hire his boat. It is known that

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 20 - \frac{980}{t^2},\ 0 < t \le 12$$

where P is the cost per hour, in Norwegian krone (NOK), that the customer is charged and t is the time, in hours, spent on the boat.

The cost per hour has a local minimum when the boat is hired for h hours.

(a) Find the value of h. [2]

Sandra hired Giles' boat for 5 hours and was charged NOK 328 per hour. Yvonne hires Giles' boat for 7 hours.

(b) Show that the cost per hour for Yvonne is NOK 312. [6]

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References:

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